

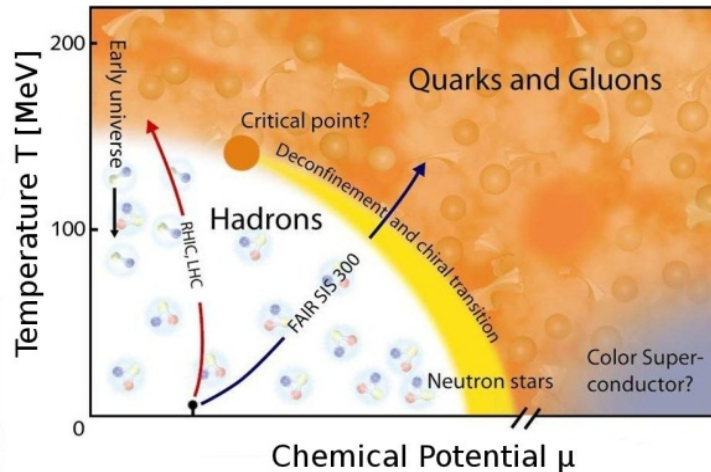
Correlation functions in Landau-gauge QCD

**RBRC Workshop
QCD in Finite Temperature
and Heavy Ion Collisions**

Nils Strodthoff, LBNL
In collaboration with A. Cyrol, M. Mitter and J. Pawlowski

QCD Phase Structure

Aim: Quantitative understanding of the **phase structure of QCD**



➤ adapted from GSI

Phase structure at large chemical potentials largely unknown due to **sign problem** in lattice QCD...

Quantitative understanding requires **first-principle approaches**

- Lattice QCD
- Functional approaches
 - ✓ Complementary to the lattice
 - ✓ No sign problem
 - ✓ Calculation of realtime observables

Functional Approaches

- Dyson-Schwinger equations (DSE)
- n-particle irreducible methods (nPI)
- **Functional Renormalization Group (FRG)**

use relations between off-shell Green's functions

$$\partial_t \longrightarrow^{-1} =$$

The diagrammatic equation shows the flow of the inverse propagator. It consists of the following terms:

- A self-energy insertion on the propagator (a horizontal line with a cross on it, and a loop above it).
- A tadpole diagram (a horizontal line with a cross on it, and a loop above it, with a vertical line connecting the loop to the horizontal line).
- A loop diagram (a horizontal line with a cross on it, and a loop above it, with a vertical line connecting the loop to the horizontal line).
- A vertex correction diagram (a horizontal line with a cross on it, and a loop above it, with a vertical line connecting the loop to the horizontal line).

Functional Approaches

- Dyson-Schwinger equations (DSE)
- n-particle irreducible methods (nPI)
- **Functional Renormalization Group (FRG)**

use relations between off-shell Green's functions

$$\partial_t \longrightarrow^{-1} =$$

The diagram shows the flow equation for the inverse propagator. The left-hand side is $\partial_t \longrightarrow^{-1}$. The right-hand side is a sum of diagrams: a self-energy loop (wavy line), a tadpole diagram (wavy line with a cross), a ghost loop (dashed line), a ghost tadpole (dashed line with a cross), and a four-point vertex diagram (circle with a cross). The diagrams are connected by plus and minus signs, and a factor of $\frac{1}{2}$ is present for the ghost loop diagram.

Top-down approach: **fQCD collaboration**

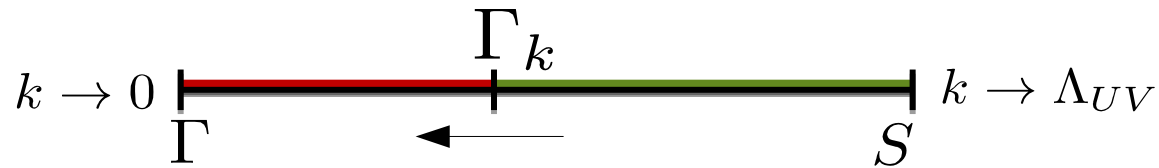
*J. Braun, L. Corell, A. K. Cyrol, W.-J. Fu, M. Leonhardt, M. Mitter,
J. M. Pawłowski, M. Pospiech, F. Rennecke, N. Strodthoff, N. Wink*

Quantitative continuum approach to QCD in the FRG framework

- ✓ No phenomenological input
- ✓ Input parameters: fundamental parameters of QCD

Functional RG for QCD

Spirit of **Wilson RG**: Calculate full quantum effective action by integrating fluctuations with momentum k



Functional Renormalization Group (FRG)

Master equation:

$$k \partial_k \Gamma_k = \frac{1}{2} \left(\text{Gluon fluctuations} - \text{Ghost fluctuations} - \text{Quark fluctuations} \right)$$

Free energy/
Grand potential

IR-Regulator

full field- and momentum-
dependent propagators

Gluon
fluctuations

Ghost
fluctuations

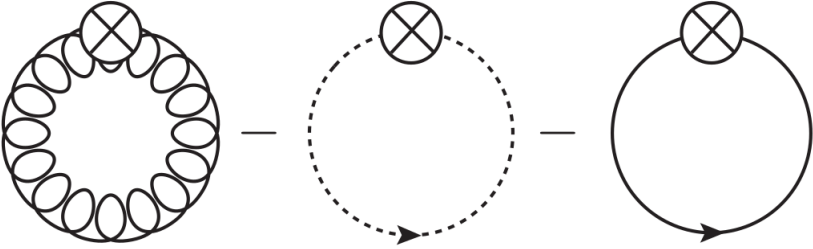
Quark
fluctuations

+flow equations for n-point functions via functional differentiation

Dynamical Hadronization

$$\frac{d\Gamma_k}{dt} = \frac{1}{2}$$

$t = \log(k/\Lambda)$



The diagram shows three Feynman diagrams arranged horizontally, separated by minus signs. Each diagram has a vertex at the top, represented by a circle with an 'X' inside. The first diagram is a circle composed of many small loops, representing gluon fluctuations. The second diagram is a dashed circle with an arrow pointing clockwise, representing ghost fluctuations. The third diagram is a solid circle with an arrow pointing clockwise, representing quark fluctuations.

Free energy/
Grand potential Gluon
fluctuations Ghost
fluctuations Quark
fluctuations

Dynamical Hadronization

$$\frac{d\Gamma_k}{dt} = \frac{1}{2} \left(\text{Free energy/Grand potential} - \text{Gluon fluctuations} - \text{Ghost fluctuations} - \text{Quark fluctuations} + \frac{1}{2} \text{Hadronic fluctuations here: } (\sigma, \pi) \right)$$

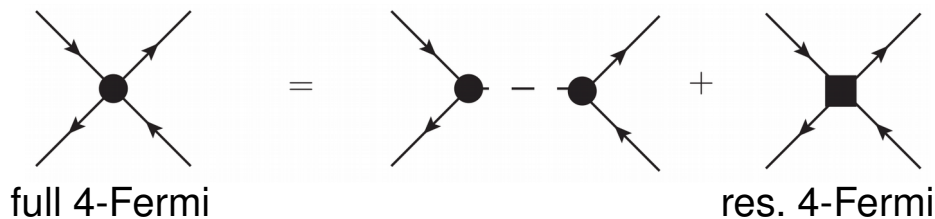
$t = \log(k/\Lambda)$

Free energy/
Grand potential Gluon fluctuations Ghost fluctuations Quark fluctuations Hadronic fluctuations here: (σ, π)

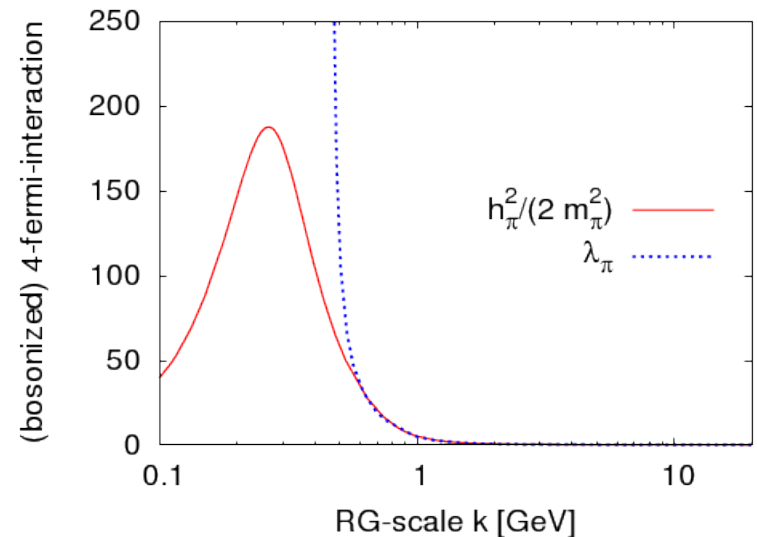
Dynamical hadronization

Store resonant 4-Fermi structures in terms of effective mesonic interactions

➤ Gies, Wetterich Phys.Rev. **D65** (2002) 065001



Efficient bookkeeping
no double counting



✓ Effective models incorporated

➤ cf. talks of J. Eser and Z. Szép

- initial conditions determined by QCD-flows

Vertex Expansion in QCD

Perturbative relevance counting no longer valid:

- Requires non-perturbative expansion schemes
- Here: **Vertex expansion** in terms of 1PI vertex functions

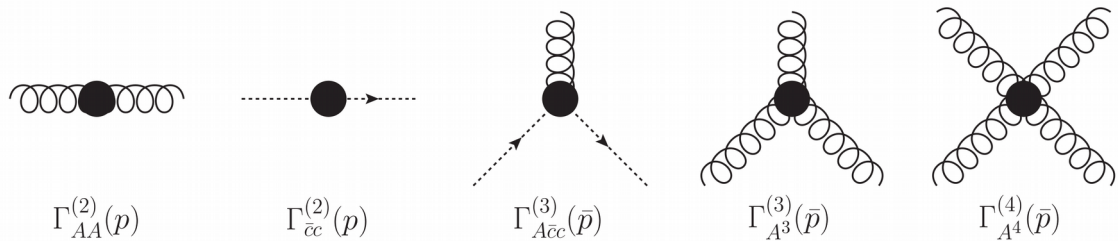
$$\Gamma[\Phi] = \sum_n \int_{p_1, \dots, p_{n-1}} \Gamma_{\Phi_1 \dots \Phi_n}^{(n)}(p_1, \dots, p_{n-1}) \Phi^1(p_1) \cdots \Phi^n(-p_1 - \cdots - p_{n-1})$$

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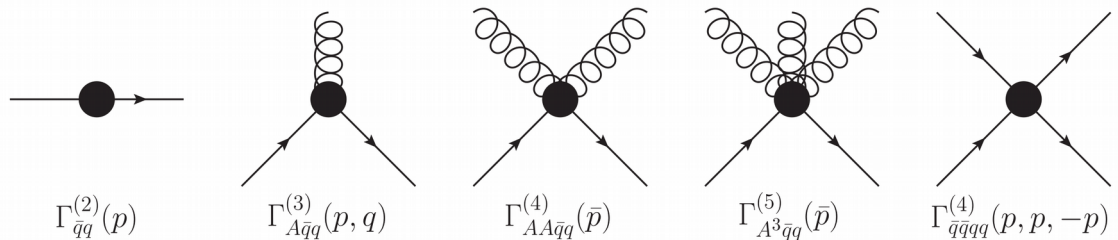
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classical tensor

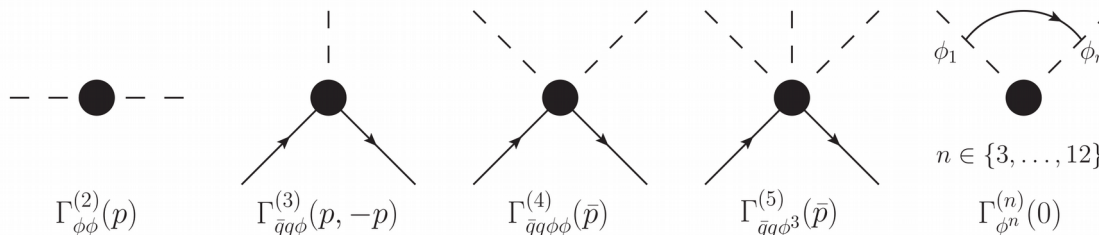
classical tensor

...aiming at apparent convergence.



$\bar{q} \not{D}^n q$ complete, $n \leq 3$

mom.-ind. tensors



$\phi \in \{\sigma, \pi\}$

“classical” tensor “classical” tensor “classical” tensor

Towards 1-Click QCD

e.g. quark-gluon vertex equation:

$$k \partial_k \text{ (quark-gluon vertex) } = - \text{ (triangle diagram) } - \text{ (triangle diagram with cross) } + \text{ (box diagram) } + \text{ (loop diagram) } + \text{ (loop diagram with cross) } - \frac{1}{2} \text{ (loop diagram) } - 2 \text{ (triangle diagram) } - \text{ (triangle diagram with cross) } + \text{ perm.}$$

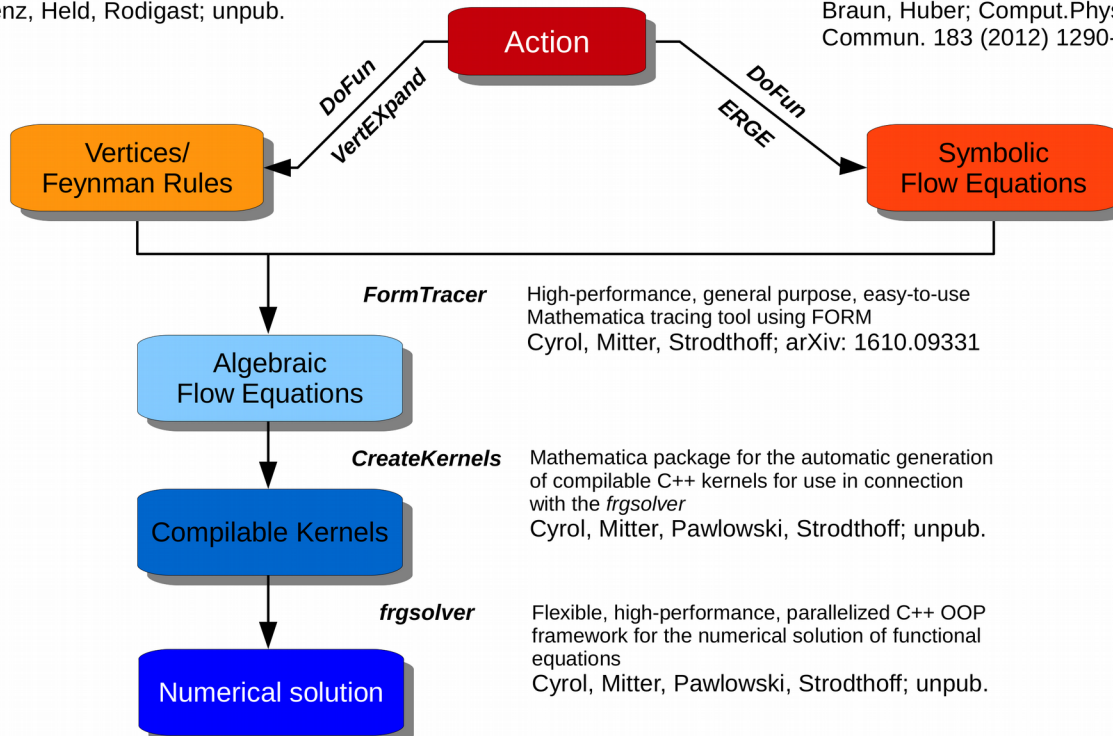
self-consistent solution

VertEXpand

Mathematica package for the derivation of vertices from a given action using FORM
Denz, Held, Rodigast; unpub.

DoFun

Mathematica package for the derivation of functional equations
Braun, Huber; Comput.Phys. Commun. 183 (2012) 1290-1320



Requirement for dedicated computer-algebraic and numerical tools

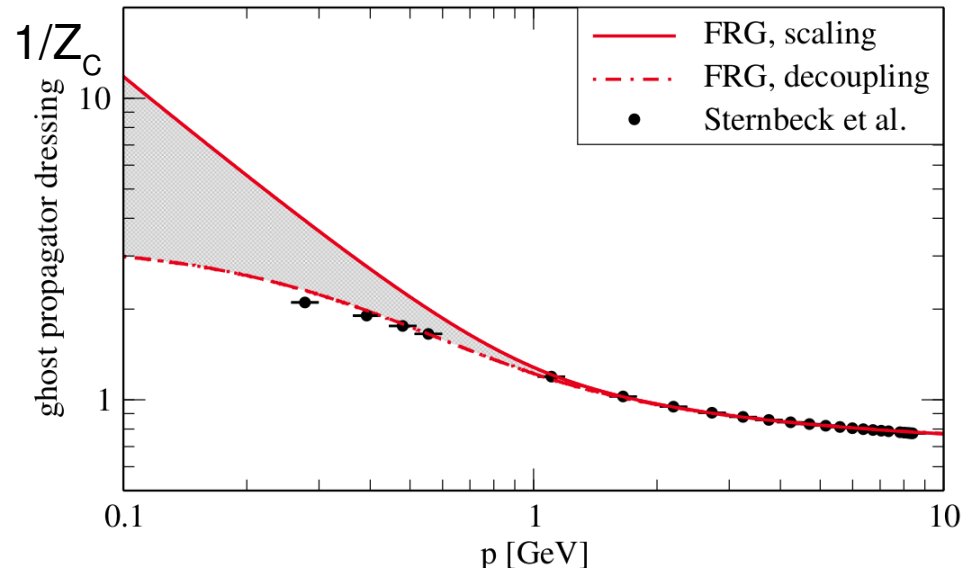
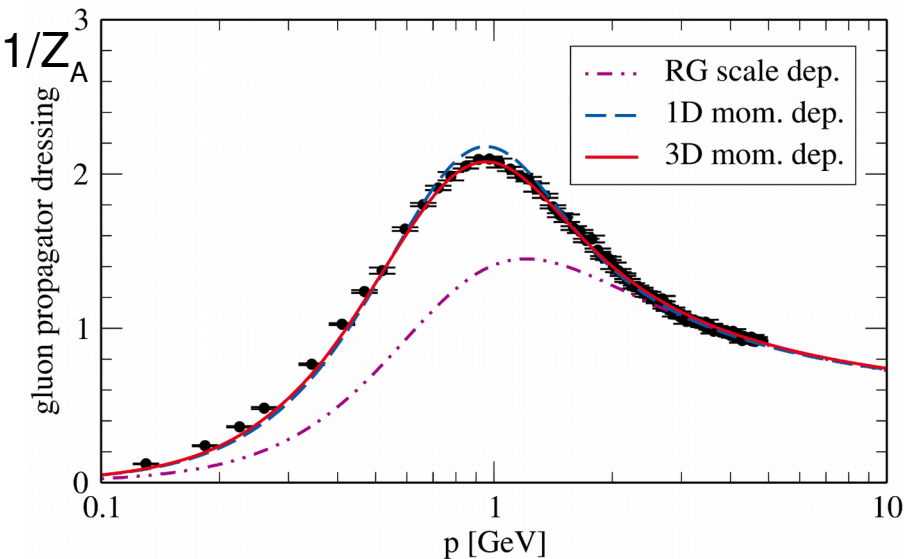
YM Propagators

Self-consistent solution of the system of transversal 2-,3- and 4-point functions

- Cyrol, Fister, Mitter, Pawłowski, NSt Phys.Rev. **D94** (2016) no.5, 054005

$$[\Gamma_{A^2}^{(2)}]_{ab}^{\mu\nu}(p) = Z_A(p)p^2\Pi_T^{\mu\nu}(p)\delta_{ab}$$

$$[\Gamma_{\bar{c}c}^{(2)}]_{ab}(p) = Z_c(p)p^2\delta_{ab}$$

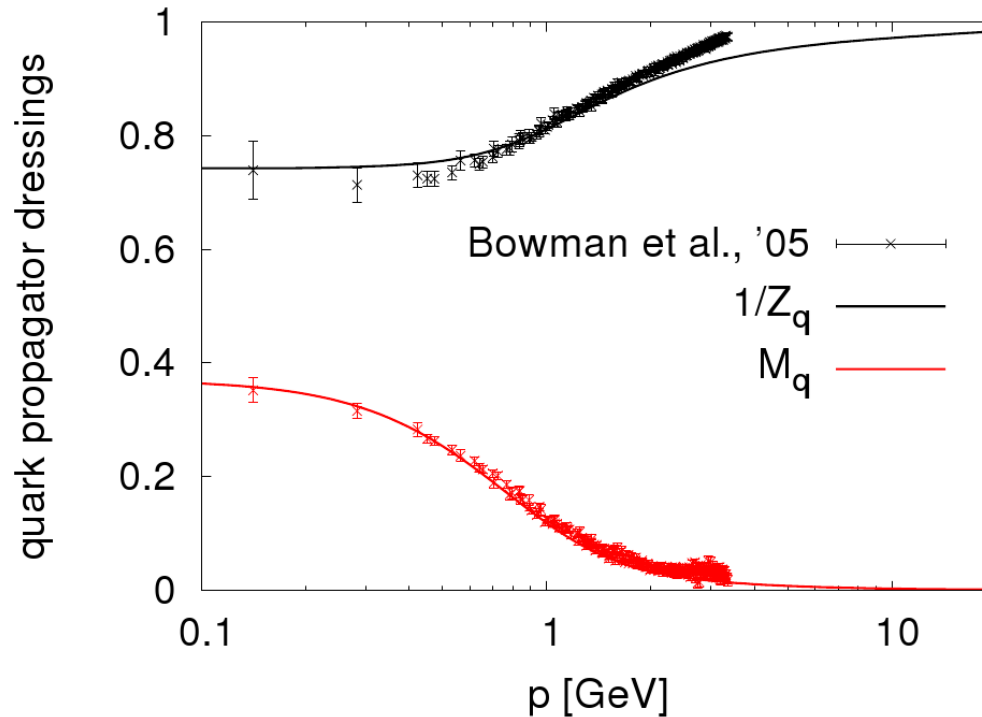


- Confinement from correlation functions:
 - Fister, Pawłowski, Phys.Rev. **D88** (2013) 045010
 - Braun, Gies, Pawłowski, Phys.Lett. **B684** (2010) 262-267
- Finite T results in preparation
 - Cyrol, Mitter, Pawłowski, NSt in prep

Quenched Quark Propagator

From the full matter system using quenched gluon propagator as only input

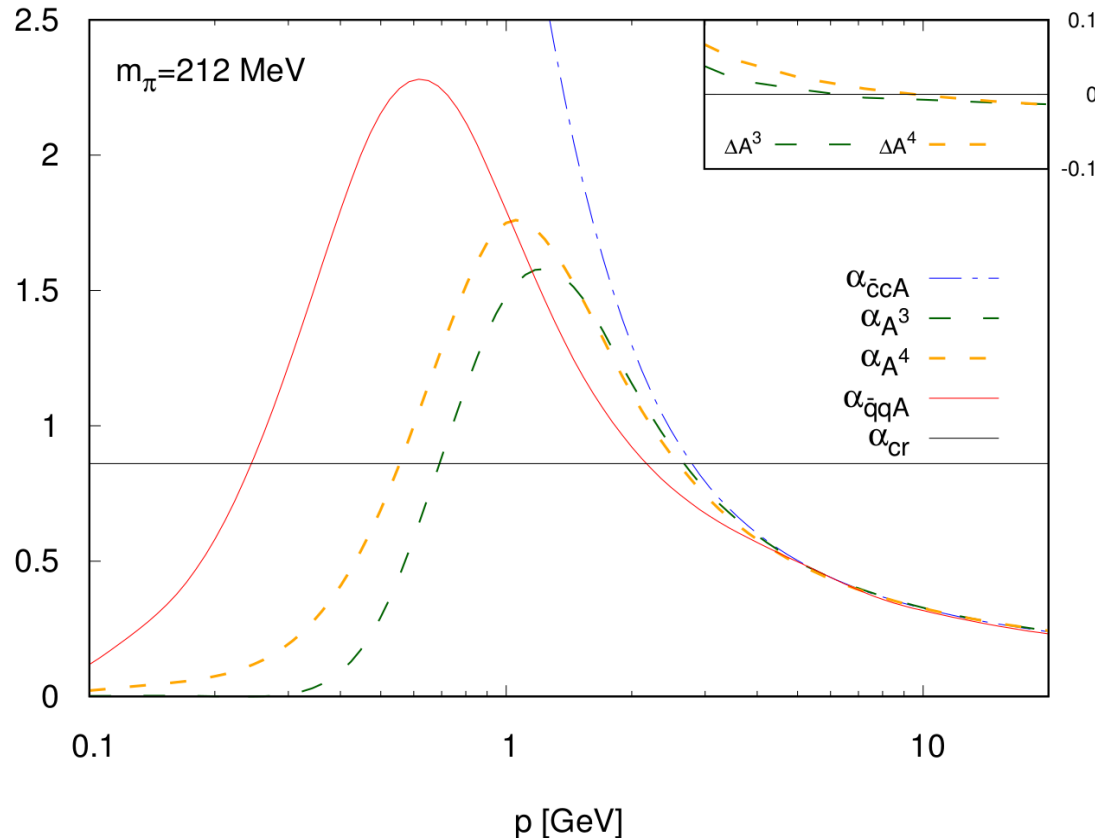
$$\Gamma_{\bar{q}q}^{(2)}(p) = Z_q(p)(i\not{p} + M_q(p))$$



Very good agreement
with (quenched)
lattice results!

➤ Mitter, Pawłowski, NSt Phys.Rev. **D91** (2015) 054035

Running Couplings & STI



$$\alpha_{\bar{c}cA}(\bar{p}) = \frac{1}{4\pi} \frac{\left(\lambda_{\bar{c}cA}^{(1)}(\bar{p})\right)^2}{Z_A(\bar{p}) Z_c^2(\bar{p})},$$

$$\alpha_{A^3}(\bar{p}) = \frac{1}{4\pi} \frac{\left(\lambda_{A^3}^{(1)}(\bar{p})\right)^2}{Z_A^3(\bar{p})},$$

$$\alpha_{A^4}(\bar{p}) = \frac{1}{4\pi} \frac{\lambda_{A^4}^{(1)}(\bar{p})}{Z_A^2(\bar{p})}.$$

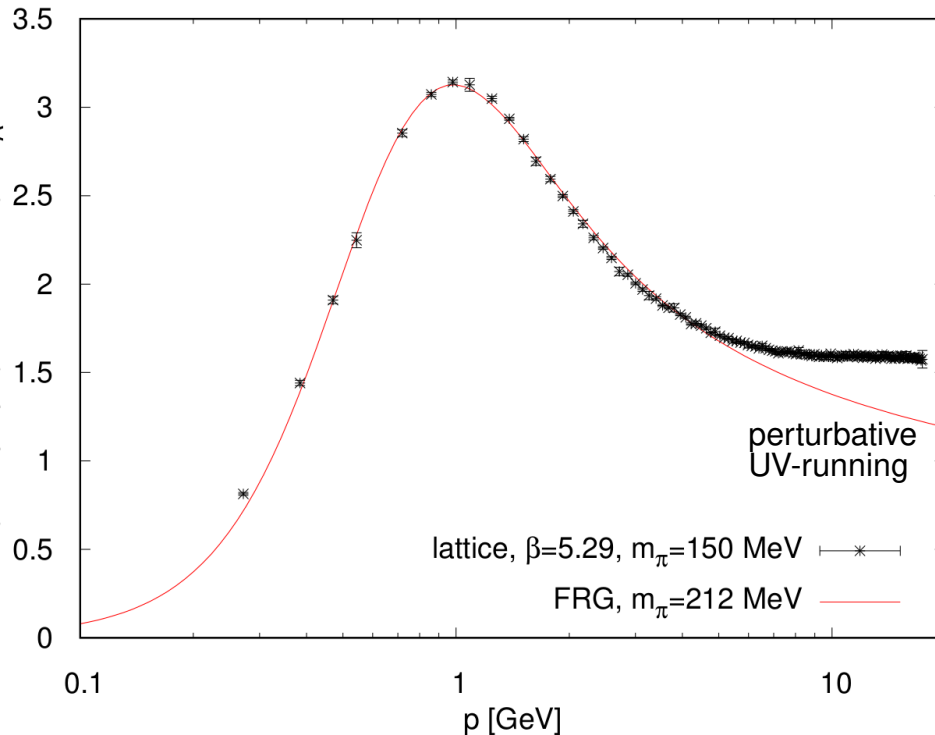
$$\alpha_{\bar{q}qA}(\bar{p}) = \frac{1}{4\pi} \frac{\left(\lambda_{\bar{q}qA}^{(1)}(\bar{p})\right)^2}{Z_A(\bar{p}) Z_q^2(\bar{p})}.$$

- Chiral symmetry breaking **very sensitive to correct semi-pert. running**
- Present solution: constrain classical tensor structure by **Slavnov-Taylor identity** in the semi-perturbative regime $\bar{p} \geq \Lambda_{\text{STI}} \rightarrow$ Davydychev, P. Osland, and L. Saks (2001)

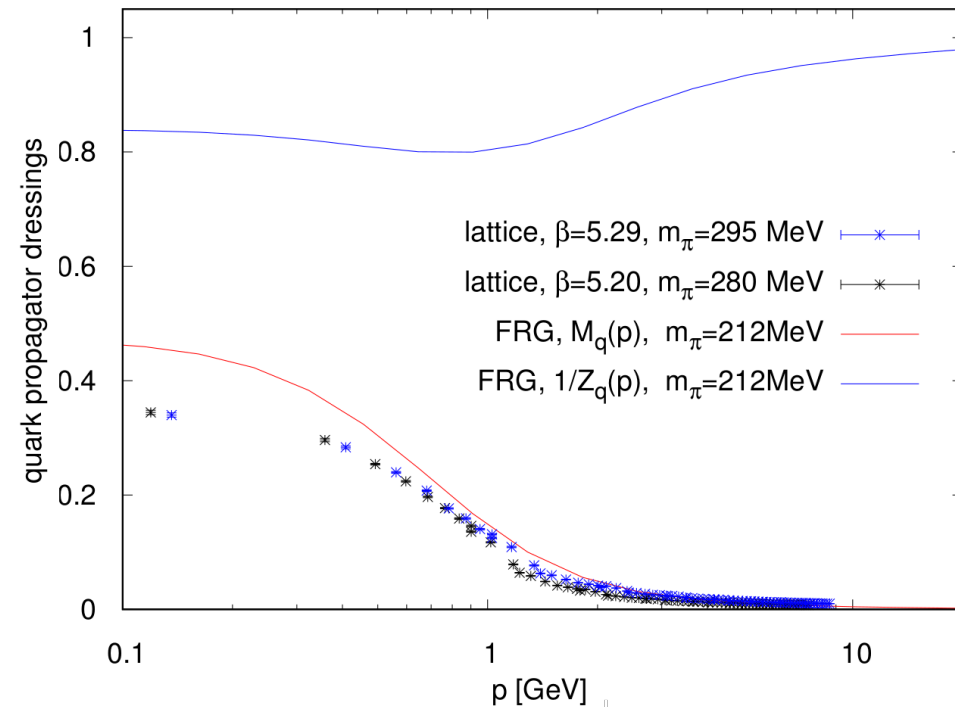
$$\lambda_{\bar{q}qA}^{(9)}(\bar{p}) = \frac{Z_q(\bar{p})}{Z_c(\bar{p})} \left[\lambda_{cqQ_q}^{(1)}(\bar{p}) - \frac{3}{2} \bar{p}^2 \lambda_{cqQ_q}^{(4)}(\bar{p}) \right]$$

Unquenched Propagators

$$[\Gamma_{A^2}^{(2)}]_{ab}^{\mu\nu}(p) = Z_A(p) p^2 \Pi_T^{\mu\nu}(p) \delta_{ab}$$



$$\Gamma_{\bar{q}q}^{(2)}(p) = Z_q(p)(i\not{p} + M_q(p))$$

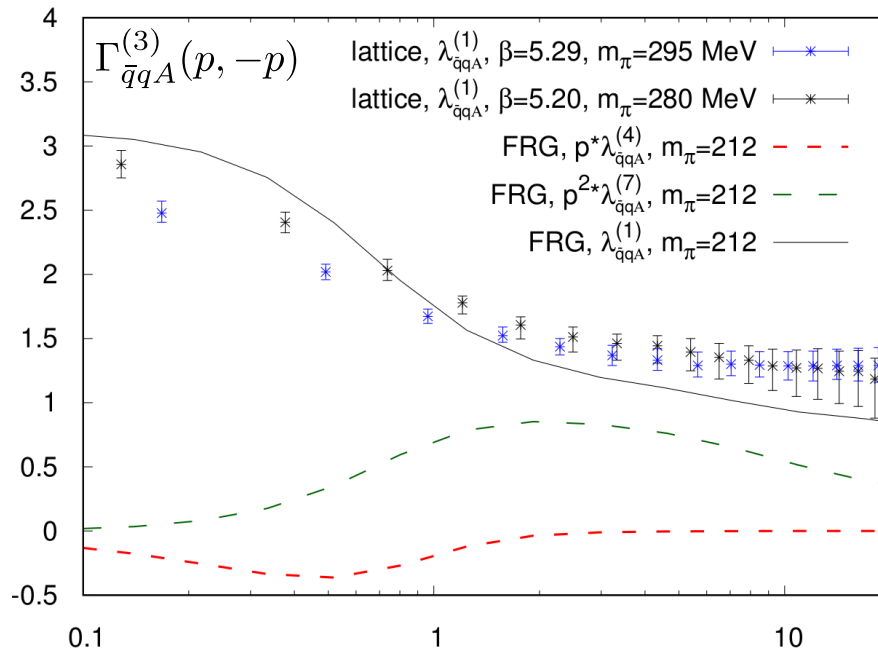


- Cyrol, Mitter, Pawłowski, NS in prep
- Lattice: Sternbeck et al
PoS LATTICE2012, 243 (2012)

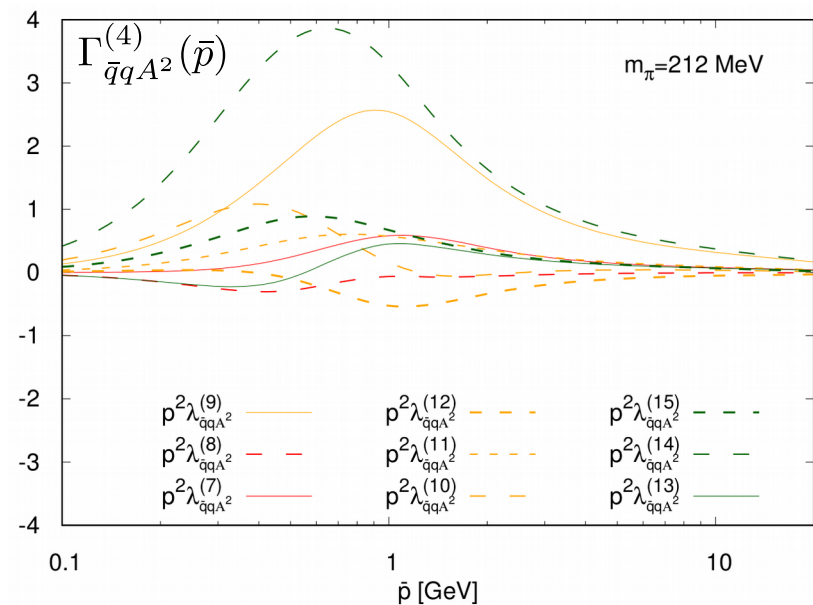
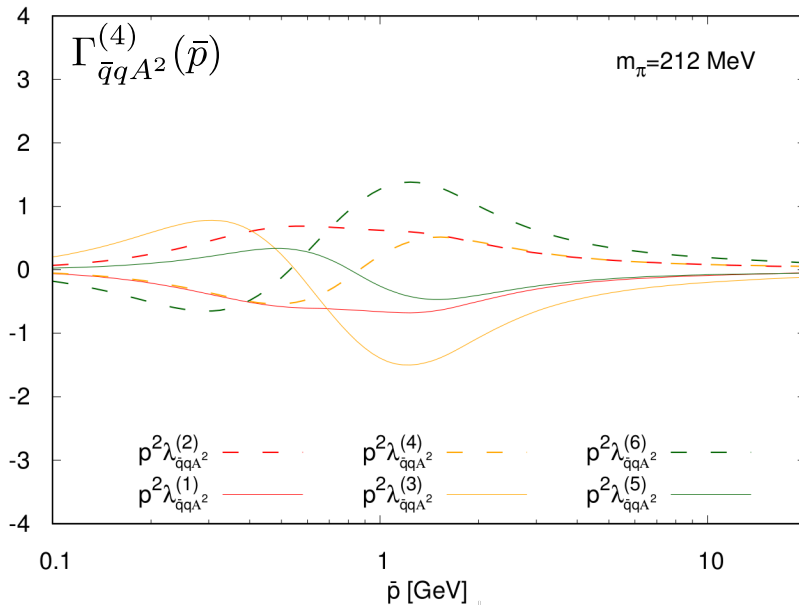
- Cyrol, Mitter, Pawłowski, NS in prep
- Lattice: Oliveira et al 1605.09632

Still to be done: proper scale-matching to the lattice (curvature vs. pole mass)

Quark-Gluon Interactions



- Three quantitatively important tensor structures
 ch. sym.: $\mathcal{T}_{\bar{q}qA}^{(1)} = -i\gamma^\mu$
 $\mathcal{T}_{\bar{q}qA}^{(7)} = -\frac{i}{2}[\not{p}, \not{q}]\gamma^\mu$
 ch. sym. br.: $\mathcal{T}_{\bar{q}qA}^{(4)} = (\not{p} + \not{q})\gamma^\mu$
- Full (3d) momentum resolution** required for quantitative accuracy
- First direct calculation of the 2-quark-2-gluon vertex



Summary

FQCD, a quantitative continuum approach to QCD

- ✓ Quenched QCD, YM Theory,...
- Unquenched QCD as a prerequisite for finite T and μ

Unquenched QCD

- First self-consistent solution of the full system
- Correct semi-perturbative running crucial for quantitative accuracy

Stay tuned...

- Finite temperature (YM)
- Directly calculated spectral functions
 - NSt 1611.05036
 - Pawłowski, NSt Phys.Rev. **D92** (2015) 9, 094009
 - Tripolt, NSt, von Smekal, Wambach Phys.Rev. **D89** (2014) 034010...
- Transport coefficients
 - Christiansen, Haas, Pawłowski, NSt PRL **115** (2015) 11, 112002
- Bound state properties
- ...

• Thank you for your attention!

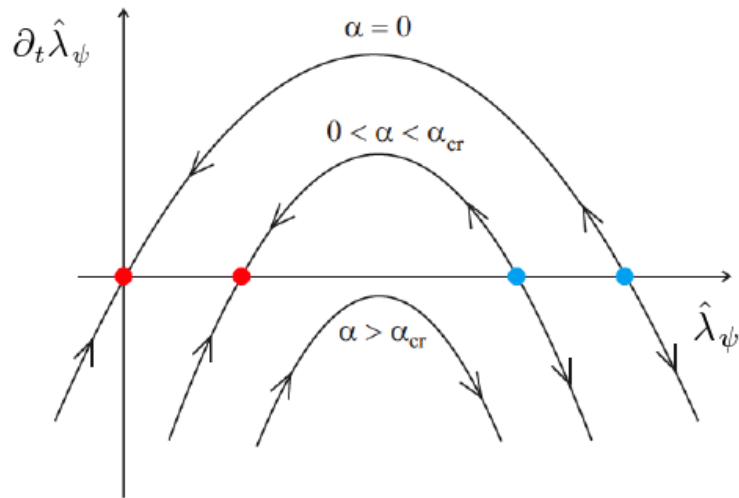
Backup

...

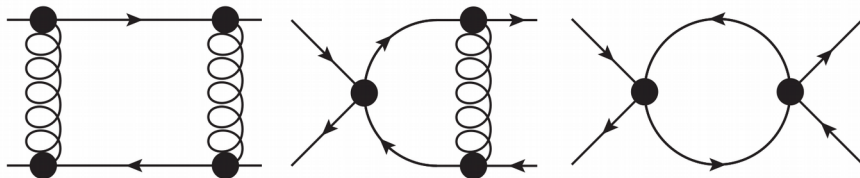
Chiral symmetry breaking

χ SB \leftrightarrow resonance in 4-quark interaction (pion pole)

β -function:



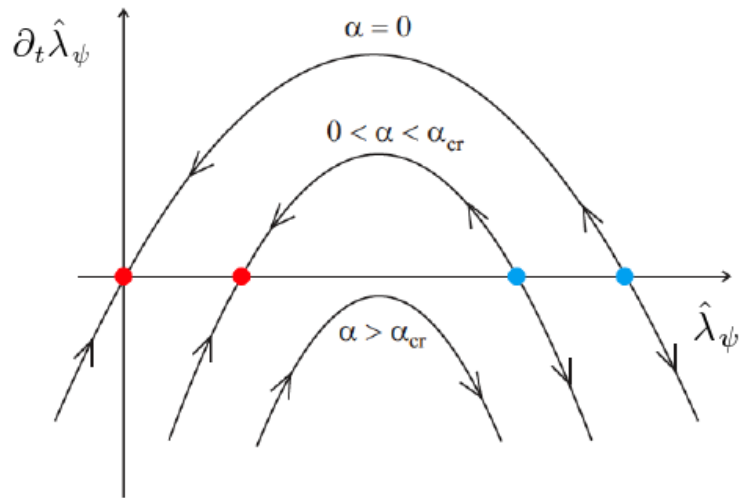
$$k \partial_k \hat{\lambda}_\psi = (d-2) \hat{\lambda}_\psi - a \hat{\lambda}_\psi^2 - b \hat{\lambda}_\psi g^2 - c g^4$$



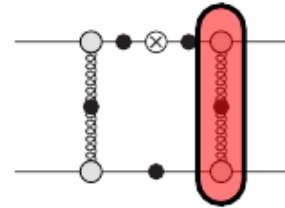
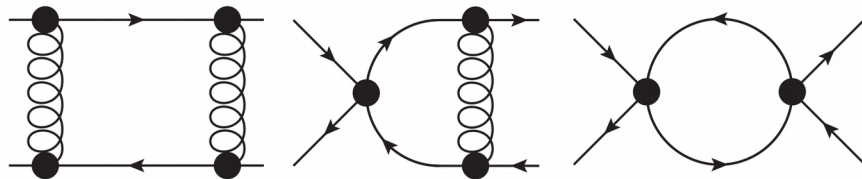
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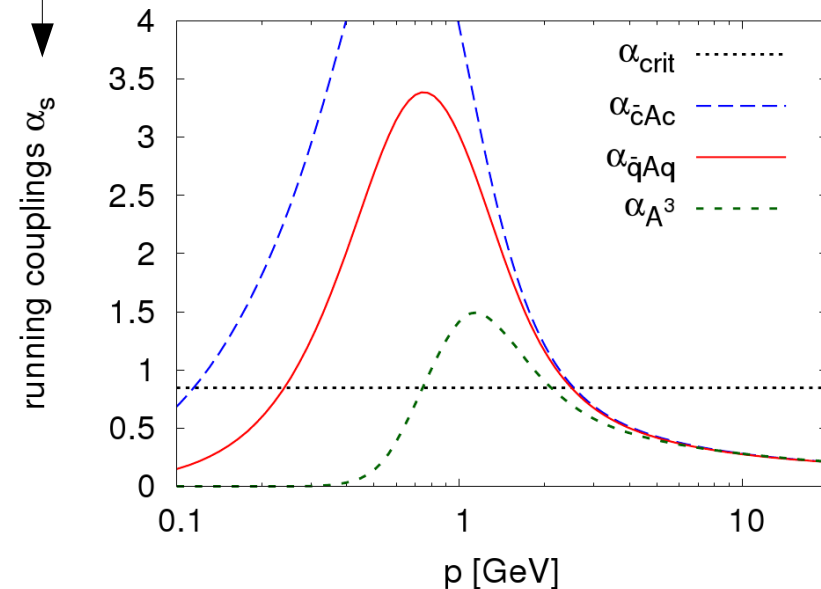
$$k \partial_k \hat{\lambda}_\psi = (d-2) \hat{\lambda}_\psi - a \hat{\lambda}_\psi^2 - b \hat{\lambda}_\psi g^2 - c g^4$$



$$\alpha_{\bar{c}Ac}(p) = \frac{Z_{\bar{c}Ac}^2(\bar{p})}{4\pi Z_A(p) Z_c^2(p)}$$

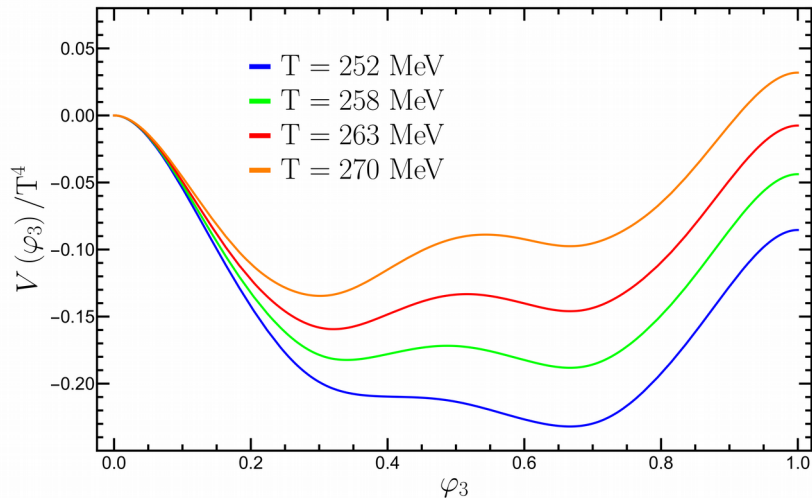
$$\alpha_{\bar{q}Aq}(p) = \frac{Z_{\bar{q}Aq}^2(\bar{p})}{4\pi Z_A(p) Z_q^2(p)}$$

$$\alpha_{A^3}(p) = \frac{Z_{A^3}^2(\bar{p})}{4\pi Z_A^3(p)}$$

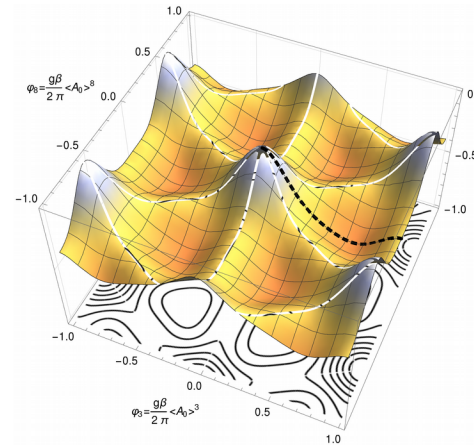


Confinement

$V(\langle A_0 \rangle)$ from YM propagators



$L(\langle A_0 \rangle)$



Confined:

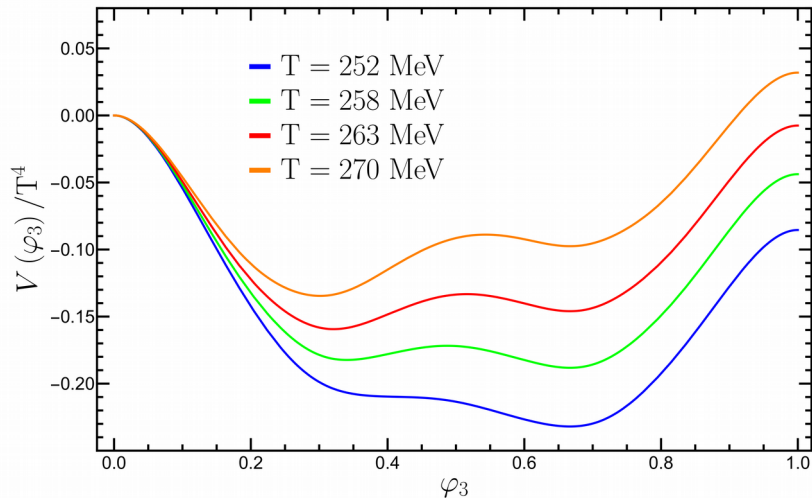
$$\bar{\varphi}_3 = \frac{2}{3}$$

$$L(\bar{\varphi}_3, 0) = 0$$

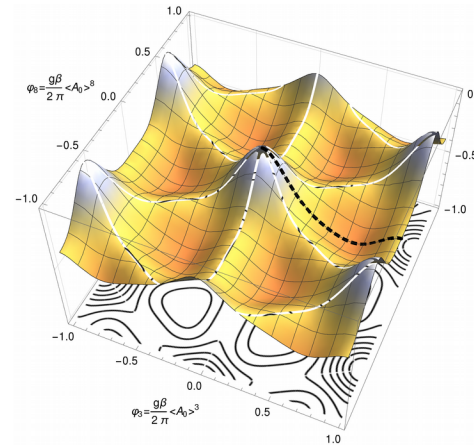
- Pawłowski, Scherzer, Strodthoff, Wink in prep.
- Herbst, Luecker, Pawłowski, (2015), 1510.03830
- Fister, Pawłowski, Phys.Rev. D88 045010 (2013)
- Braun, Gies, Pawłowski, Phys.Lett. B684 (2010)

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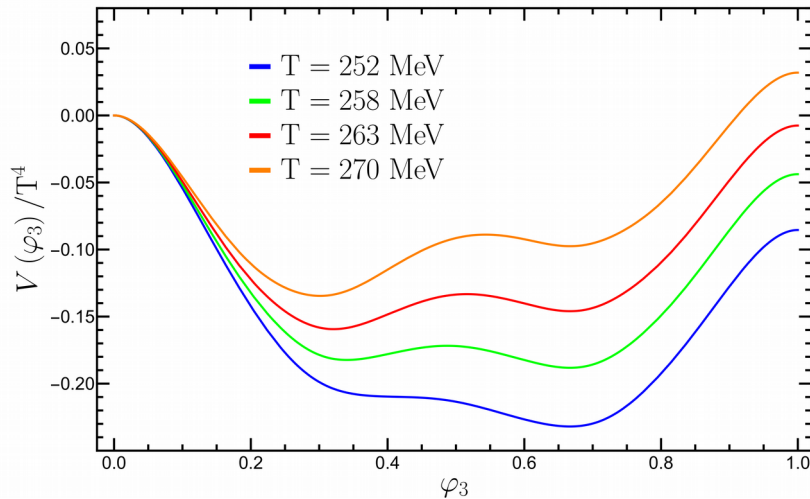
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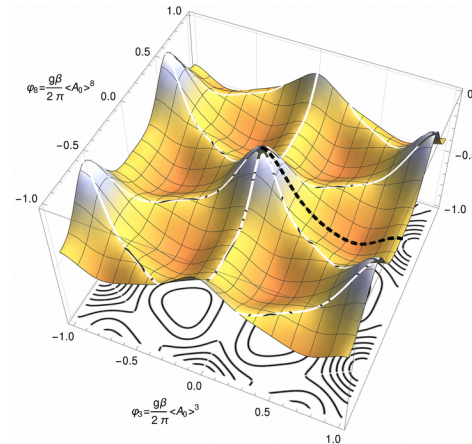
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- Fister, Pawłowski, Phys.Rev. D88 045010 (2013)
- Braun, Gies, Pawłowski, Phys.Lett. B684 (2010)

Order parameters:

$\bar{\varphi}_3$ most easily computed in
 $L(\langle A_0 \rangle)$ functional methods

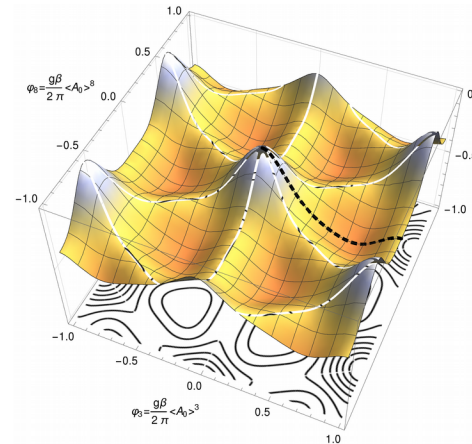
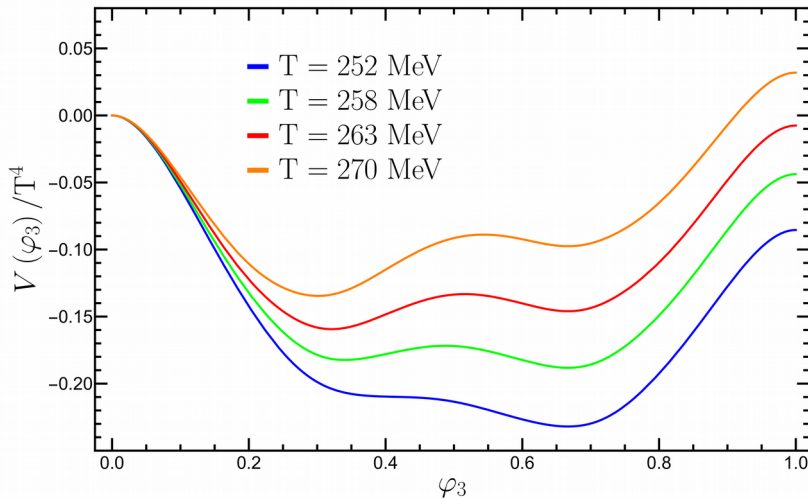
$\langle L(A_0) \rangle$ computed on the lattice;
 now also in the FRG

$$\langle L(A_0) \rangle \leq L(\langle A_0 \rangle)$$

$$\langle L(A_0) \rangle = 0 \iff L(\langle A_0 \rangle) = 0$$

Confinement

$V(\langle A_0 \rangle)$ from YM propagators



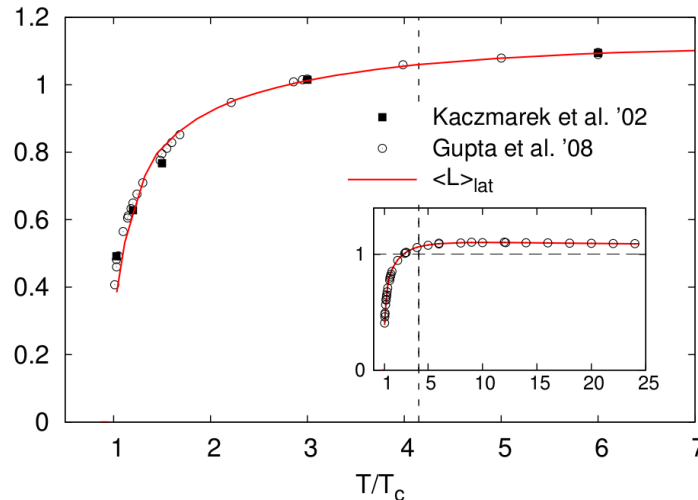
Confined:

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- Fister, Pawłowski, Phys.Rev. D88 045010 (2013)
- Braun, Gies, Pawłowski, Phys.Lett. B684 (2010)

$\langle L(A_0) \rangle$ from $L(\langle A_0 \rangle)$



Order parameters:

$\bar{\varphi}_3$ most easily computed in functional methods

$L(\langle A_0 \rangle)$

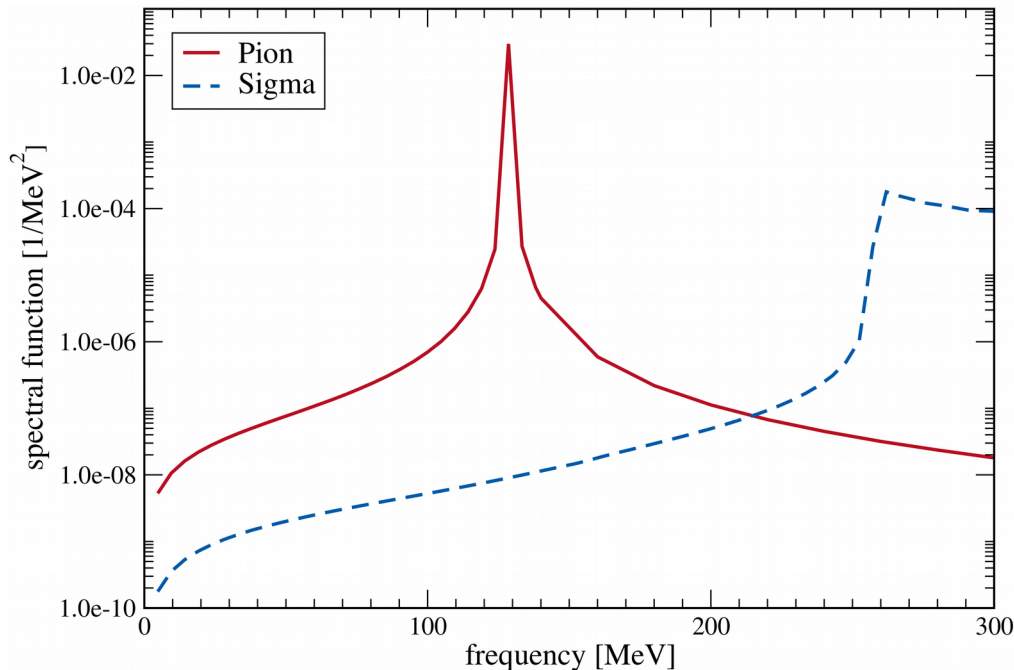
$\langle L(A_0) \rangle$ computed on the lattice; now also in the FRG

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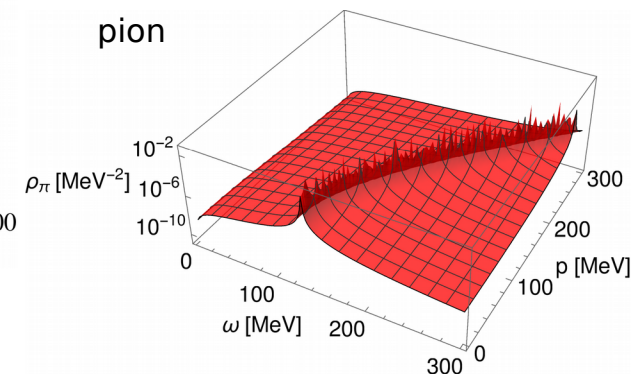
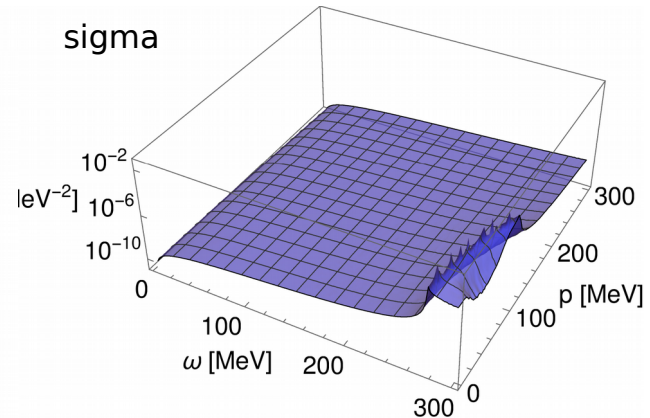
$$\langle L(A_0) \rangle = 0 \iff L(\langle A_0 \rangle) = 0$$

Spectral Functions

O(N) at T=0, full momentum dependence



➤ NSt, in prep



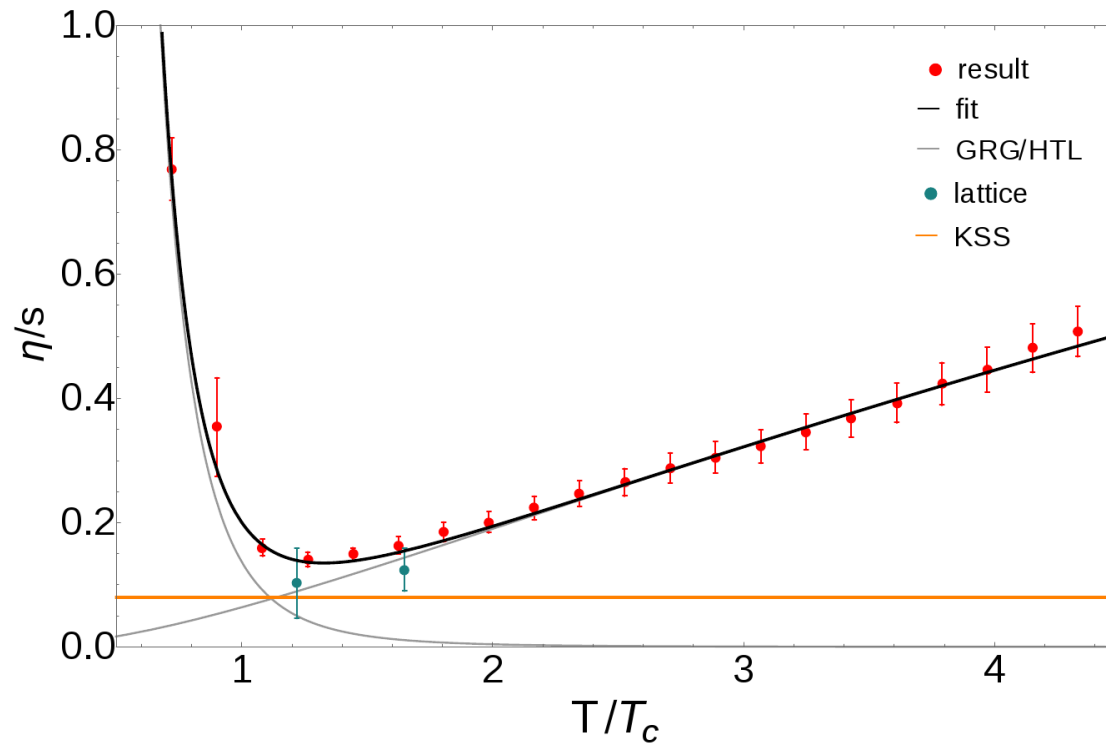
Summary

- ✓ Directly calculated spectral functions
- ✓ Tested in scalar and Yukawa models at $T, \mu > 0$
- ✓ Allows the inclusion of full momentum dependence

❑ Quark & gluon spectral functions in full QCD

- NSt 1611.05036
- Pawłowski, NSt
Phys.Rev. **D92** (2015) 9, 094009
- Tripolt, NSt, von Smekal, Wambach
Phys.Rev. **D89** (2014) 034010
- Kamikado, NSt, von Smekal, Wambach
Eur.Phys.J. **C74** (2014) 2806

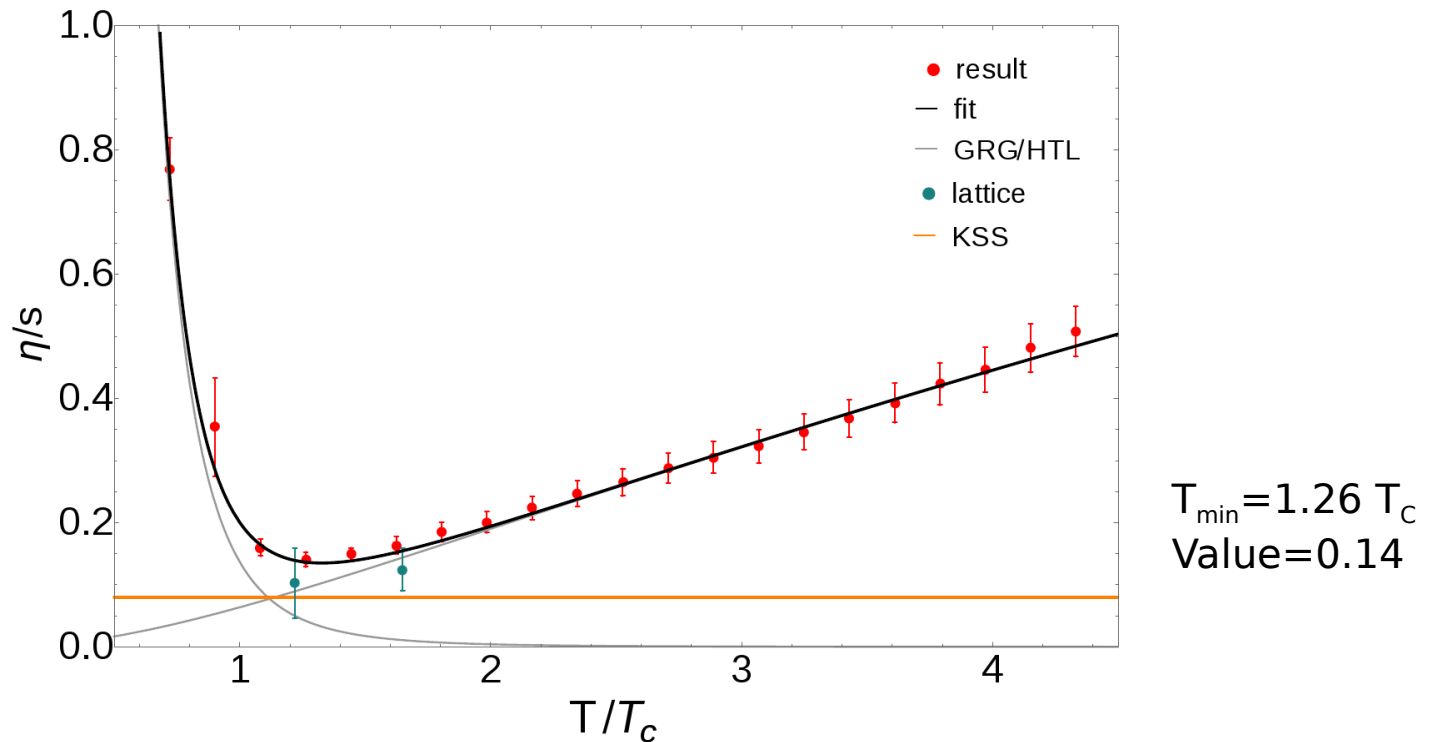
η/s in Yang-Mills Theory



$T_{\min} = 1.26 T_c$
Value = 0.14

➤ Christiansen, Haas, Pawłowski, NSt PRL **115** (2015) 11, 112002

η/s in Yang-Mills Theory



➤ Christiansen, Haas, Pawłowski, NSt PRL **115** (2015) 11, 112002

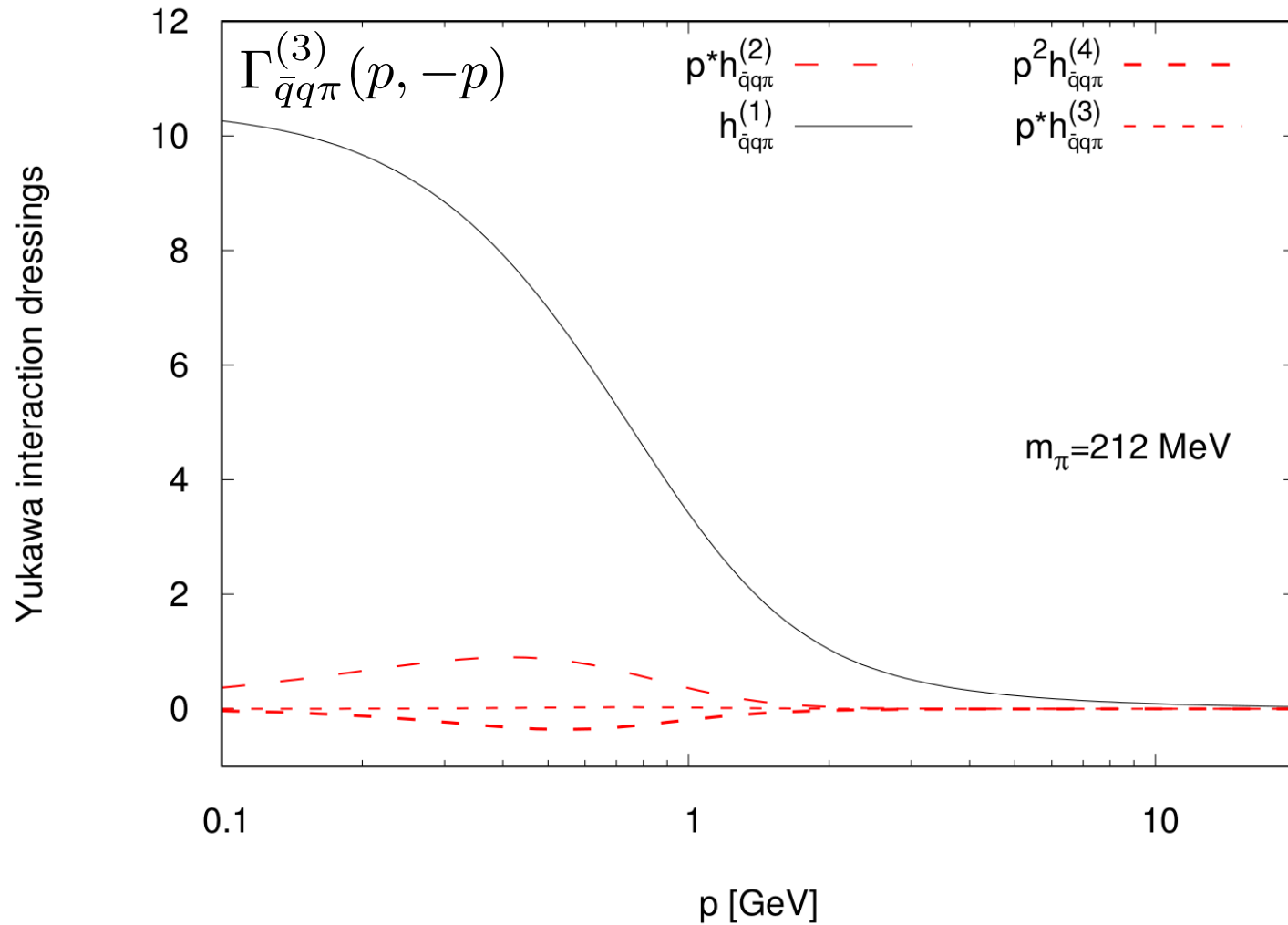
$$\text{Direct sum: } \frac{\eta}{s}(T) = \frac{a}{\alpha_s (cT/T_c)^\gamma} + \frac{b}{(T/T_c)^\delta}$$

$$\gamma = 1.6 \quad a = 0.15 \quad b = 0.14 \quad c = 0.66 \quad \delta = 5.1$$

High T: **consistent with**
HTL-resummed pert. theory (fixing γ)
 supporting quasiparticle picture

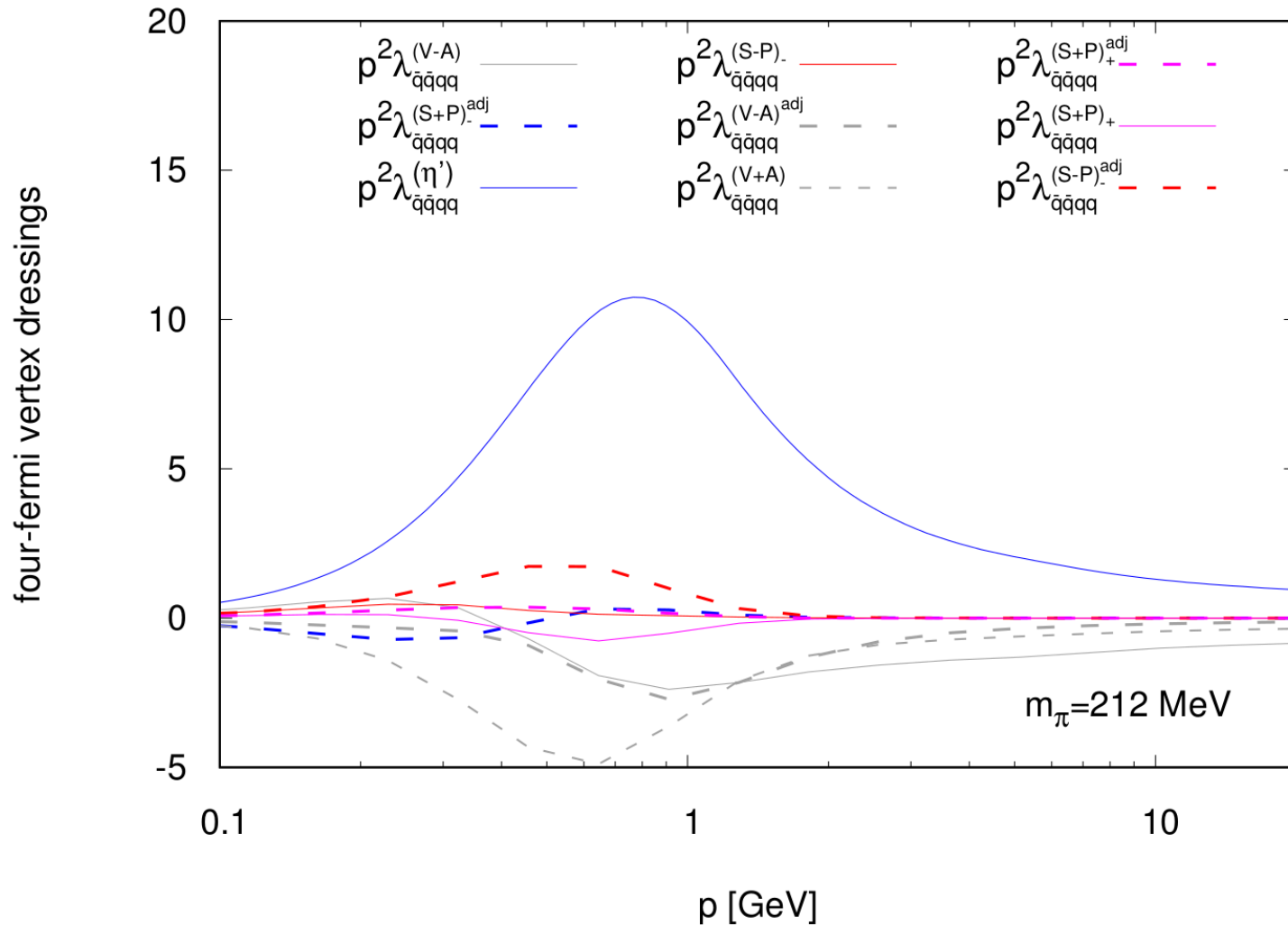
Small T: algebraic decay
glueball resonance gas

Quark-Meson Interactions



4-Fermi Interactions

$$\Gamma_{\bar{q}\bar{q}qq}^{(4)}(p, p, -p)$$



Ghost Propagator

$$\Gamma_{\bar{c}c}^{(2)}(p) = Z_c(p)p^2$$

